Introduction :-

→ In Thermodynamics deals with amount of heat transfer as a system undergoes a process from one Equilibrium state to another and Makes no reference to how long the process will take.

→ But Engineering we often intrested in the rate of heat transfer which is the topic of the science of Heat transfer.

→ In this chapter we lay the foundation for much of the material treated in the text. we do so by Raising several questions.

→ What is Heat transfer
→ How is Heat transfered
→ Why is it important

1) What is Heat transfer :- Heat transfer (or Heat) is the thermal Energy in transit due to a spatial temparature difference.

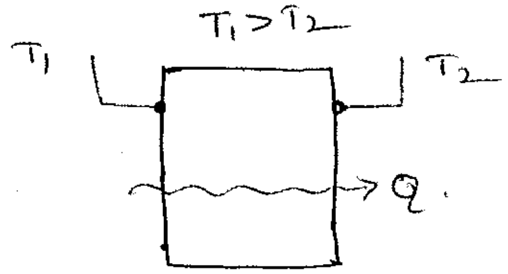
2) How is Heat transfered :- we Refer to different types of Heat transfer process as modes. when the temparature gradient exists in a stationary medium, which may be a solid (or) a fluid, we use the terms modes of Heat transfer.

Modes of Heat transfer :-

1) conduction 2) convection 3) Radiation.

Conduction :-

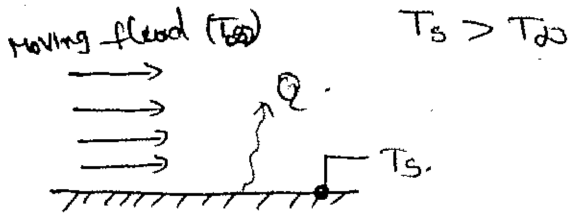
Conduction is the transfer of Energy from the More Engergitic particles of substance to the adjacent Less Energatic ones, as a result of interactions b/w. the particles.



Conduction through a solid (or) stationary fluid

Convection :-

Convection is a Mode of heat transfer b/w solid surface and the adjacent liquid (or) gases that is in motion and involves combined Effect of conduction and Fluid Motion



Convection from a surface to a moving fluid.

Radiation :-

Radiation is a an Energy Emitted by a matter in the form of Electromagnatic waves (or photons) as a Result of changes in the Electronic Configurations of the atoms (or) Molecules.

Examples :-

1) cold canned drink left in Room warms up. higher to lower temperature.

→ In TD we are Intrested in Heat ie Heat transfer

→ In H.T we deals with rates of Heat transfer

2) 90°C of body kept in Refrigirator to cool 50°C.

→ In TD only tells only it will happen, but designer need to how it will occur from 90°C to 50°C, so the subject HT deals the how long it will occur [Time factor].

Applications :-

1) Design of steam generators, condensers, and other heat exchanging equipments in power plant.
2) IC Engines, Refrigirators, Air conditioning Equipments
3) Gas turbine blades, Aero space Applications etc ---

Heat transfer Mechanisms :-

→ Heat is the Form of Energy that can be transfered from one system to another as result of temp difference.

→ In HT subject heat transfer with rate we study.

→ ~~That~~ Heat can be transmitted in '3' different Modes

1) conduction
2) convection
3) Radiation.

1. Conduction :-

→ It is the transfer of Energy from the More Energitic particles of substance to adjacent Less Energatic ones as a result of interactions b/w the particles.

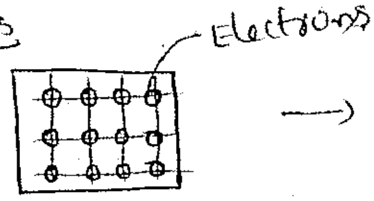
→ conduction can takes place in solids, Liquids (or) gases.

→ In solids conduction takes place Lattice Vibrations and Flow of free Electrons

→ In liquids conduction takes place Molecular collisons

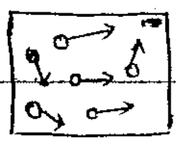
→ In gases conduction takes place Molecular Diffusions

Solids



→ 1) Lattice Vibrations
2) Flow of free Electrons

Liquids & gases



→ 1) Molecular collision
2) Molecular Diffusion.

→ The Rates of Heat ~~transfer~~ conduction through a Medium depends upon

1) Geometry of a Medium
2) Thickness of Medium
3) Material of the Medium
4) Temp difference accross the Medium

→ So, we conclude that "the Rate of Heat conduction Through a plane Layer is proportional to the the temparature difference accross the layer & The Heat transfer area, but it is Inversely proportional to the thickness of layer."

ie.,

Rate of Heat conduction $\propto$ $\dfrac{(\text{Area})(\text{temp diff})}{\text{Thickness}}$

$$\dot{Q}_{cond} \propto A \cdot \frac{\Delta T}{\Delta x}.$$

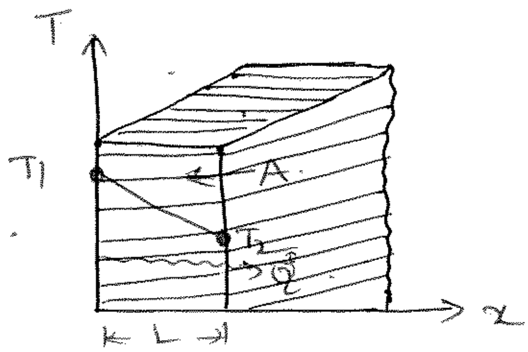
$\Delta x \to 0$ so differential Form

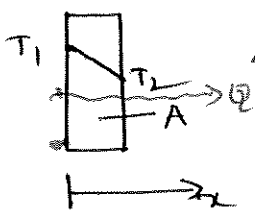
$$\boxed{\therefore \dot{Q}_{cond} = -KA \cdot \frac{dT}{dx}.}$$

↓

Fourier law of Heat conduction.

↳ Derived by J. Fourier.





Where 'K' is the thermal conductivity (or) proportional constant.

$\frac{dT}{dx}$ → Temp gradient.

→ Here "-ve" sign is indicated for positive Heat transfer, because

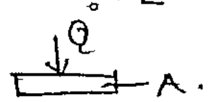
dT = final temp − initial temp

$dT = 200 - 300$

$dT = -100$

Here Temp is ↓ with ↑x so -ve sign introduced.

→ The heat transfer area 'A' Always normal [$\perp^{lr}$] to the direction of Heat transfer



Let us see what is k :-

→ we have seen that different Materials stores heat differently ie, a property called specific Heat (C_p), Means ability to store thermal Energy

Ex:-

C_p of water = 4.18 KJ/kg·K (or) KJ/kg°c.

C_p of Iron = 0.45 KJ/kg·K } both are at ROOM temp

→ If we observe that water has more Energy storage capacity, so that it cannot conduct (transfer) heat

→ As Iron it has lower energy storage capacity, so as well as it has higher thermal conductivity

ie,

$$K_{water} = 0.607 \ \omega/mK.$$

$$K_{iron} = 80.2 \ \omega/mK.$$

→ so we can say water has poor thermal conductivity relative to Iron and Excellent Medium to store Energy.

→ so Thermal conductivity is defined as "The rate of Heat transfer through a unit thickness of the material per unit area per unit temp difference."

→ ∴ From Heat conduction Equation

$$Q = KA \frac{dT}{dx} \quad - 1$$

$$\boxed{K = \frac{Q \cdot dx}{A \cdot dT}}$$

$$K = \frac{\omega \cdot m}{m^2 K}$$

$$\boxed{K = \omega/m \cdot K}$$

Imp ~~cases~~ points:-

1) ↑ K ↑ conductivity

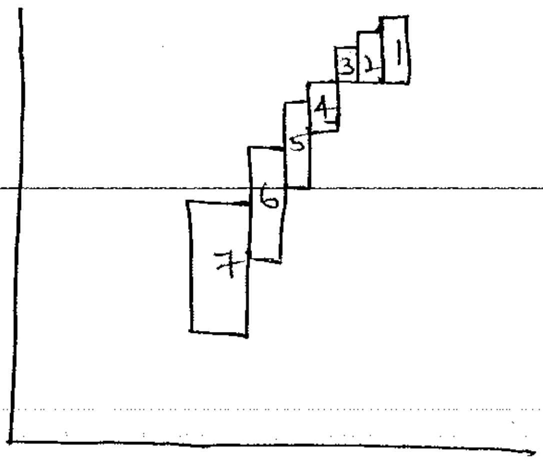
2) ↓ K ↓ conductivity, it means it has poor conductor (or) Insulator.

→ Diamond has ↑ k = 2300 w/mk, → Air and Rigid foam has ↓ k = 0.026 w/mk.

→ Al = 429 w/mk, Cu = 401 w/mk, Gold = 317 w/mk

→ Fe = 80.2 w/mk.

→ Non Metalic Crystaline k is ↑



1 → Non Metallic Crystals
2 → pure metals
3 → Metal alloys
4 → Non Metallic solids
5 → Liquids
6 → Insulators
7 → gases.

→ Thermal conductivity of Materials are vary with temperature.

Thermal Diffusivity [α] :-

→ The product "ρC_p" frequently encounted in heat transfer Analysis called heat capacity

→ It is also like wise "C_p", but the difference is; ~~"Cp" is ex~~

→ "C_p" is expressed per unit mas, as "ρC_p" expressed per unit volume.

$$C_p = \frac{J}{kg\,K}$$

kg → Mass

$$\rho C_p = \frac{kg}{m^3} \times \frac{J}{kg\,K} = \frac{J}{m^3\,K}$$

m^3 → volume

→ In transient heat conduction frequently appears thermal diffusivity ie $\underline{\alpha}$

→ $\underline{\alpha \text{ Means}}$ How fast Heat diffuses through a Material & defined as

$$\boxed{\alpha = \frac{\text{Heat Conduction}}{\text{Heat store}} = \frac{k}{\rho C_p}}$$

$$= \frac{W}{mK} \times \frac{m^3 K}{J}$$

$$= \frac{\cancel{J}}{s\,\cancel{m}\cancel{K}} \times \frac{m^{\cancel{3}}\cancel{K}}{\cancel{J}}$$

$$= \boxed{m^2/s.}$$

→ If $k \uparrow$ & $\rho C_p \downarrow \Rightarrow \alpha \uparrow$.

Conventional Questions :-

1) The wall of an Industrial Furnace is constructed from 0.15m thick fireclay brick having a thermal conductivity of 1.17 W/m·K. Measurements made during steady-state operation reveal temperatures are 1400 and 1150K at the inner and outer surfaces, respectively. What is the rate of heat loss through a wall that is 0.5m X 1.2m on a side?

Data

Thickness (L) or (dx) = 0.15 m

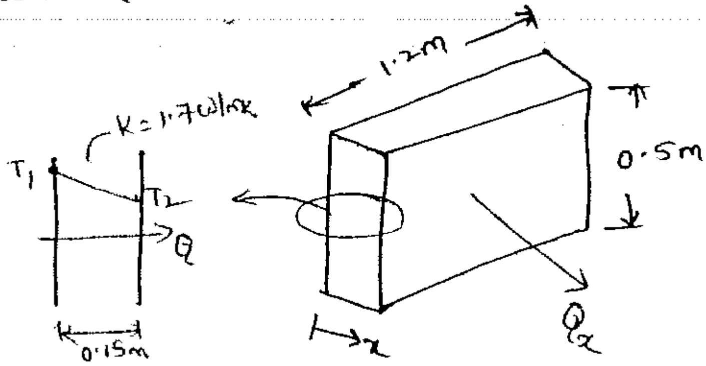
Thermal conductivity (K) = 1.17 W/m·K.

T_1 = 1400K , T_2 = 1150 K.

Surface Area of wall (A) = 0.5m X 1.2m.

Find :-

1) Heat Loss =?



Assumptions :-

1) steady-state ~~conduction~~ condition

2) one-Dimensional conduction through the wall

3) constant thermal conductivity.

Sol

$$\dot{Q} = -KA\frac{dT}{dx}$$

$$= -1.17 \times 0.5 \times 1.2 \times \frac{(1150 - 1400)}{0.15}$$

$$\dot{Q} = 1700\ W.$$

2) The roof of an electrically heated home 6m long, 8m wide and 0.25m thick & is made of flat layer of concrete whose thermal conductivity is $K = 0.8\ W/m\cdot K$. The temp of the Inner and outer surfaces of the roof one night are measured to 15°C and 4°C, respectively, for a period of 10 hours. Determine 1) The Rate of Heat Loss through the roof that night and 2) The cost of that heat Loss to the home owner if the cost of electricity is 5/- per kWhr.

Data :-

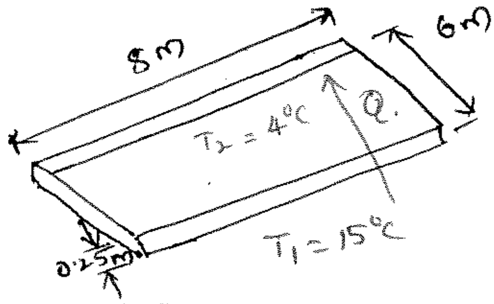
surface area of concrete ~~slab~~ Roof (A) = 6m × 8m

thickness of ~~slab~~ Roof (L) or (dx) = 0.25m.

Thermal conductivity (K) = 0.8 W/mK.

$T_1 = 15°C$, $T_2 = 4°C$.

Time (t) = 10 hrs.



find :-

1) Rate of Heat Loss ($\dot{Q}$) = ?

2) Electricity ~~Loss~~ ~~cost~~ Loss = ?

Assumptions :-

1) steady-state conditions
2) one-Dimensional conduction throughout the roof
3) constant thermal conductivity.

Sol

1) $$\dot{Q} = -KA\frac{dT}{dx}$$

$$= -0.8 \times 6 \times 8 \times \frac{(4-15)}{0.25}$$

$$Q = 1690\,W$$
$$= 1.69\,kW.$$

2) cost of Electricity

$$= 1.69 \times 10 \times 5$$
$$= \underline{\underline{84.5/-}}$$

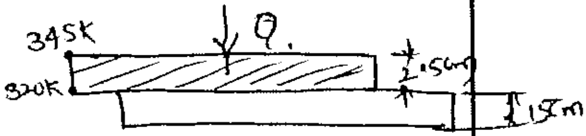
3) To Effect a bond b/w two metal plates 2.5cm and 15cm thickness, heat is uniformly applied through the thinner plate by a radiant source. The bonding Epoxy must be held at 320K for short time. when the Heat source is adjusted to take a steady value of $43.5\ kW/m^2$, a thermocouple installed on the side of the thinner plate next to source indicates a temp of 345K. calculate the temp gradient for Heat conduction through thinner plate and thermal conductivity of its Materials

Data:-

$$L = dx = 2.5cm = 0.025m$$
$$T_1 = 320K,\ T_2 = 345K$$
$$\frac{Q}{A} = q = 43.5\ kW/m^2 = 43.5 \times 10^3\ W/m^2$$

Find:-

1) Temparature gradient $\left(\frac{dT}{dx}\right) = ?$



2) Thermal conductivity $(K) = ?$

Assumptions :-

1) steady -state condition
2) one - Dimensional conduction through the plate
3) const. Thermal conductivity.

Sol :-

1) $$\frac{dT}{dx} = \frac{T_1 - T_2}{dx}$$

$$= \frac{320 - 345}{0.025} = -1000\,°K/m.$$

2) $\frac{Q}{A} = q$ / $Q = -KA \cdot \frac{dT}{dx}$

$$\frac{Q}{A} = q = -K \cdot \frac{dT}{dx}$$

$$K = -\frac{q \cdot dx}{dT}$$

$$K = -\frac{43.5 \times 10^3 \times 0.025}{345 - 320}.$$

$$K = 43.5\ W/m°K.$$

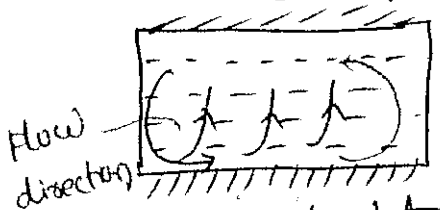
2) Convection :-

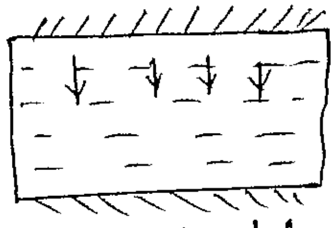
→ convection is a Mode of Heat transfer between a solid surface and adjacent Liquid (or) gas [Fluid], that is in Motion, and it involves the combine effect of conduction & Fluid Motion

→ If Any absence of Fluid Motion b/w solid surface and adjacent fluid is by pure conduction

→ Faster the fluid motion, greater the convective heat transfer.

→ The effectiveness of heat transfer by convection ~~depends~~ depends ~~on~~ largely upon the mixing motion of the fluid.





↓

This is convection

↓

There is fluid particle motion due to ~~density~~ buoyancy effect by the differences of density differences

↓

This is pure conduction.

↓

No fluid motion, because hotter particles stay at upper surface ~~side~~ only.

→ At initial heat is first transferred to the air layer adjacent to any hot block by conduction only, then it is carried away from the surface by convection.

→ The rates of convective heat transfer is directly proportional to the temp differences & surface area of the geometry, & it is expressed by Newton's Law of cooling as

$$\boxed{\dot{Q}_{conv} \propto A_s (T_s - T_\infty)}$$

$$\boxed{\dot{Q}_{conv} = h A_s (T_s - T_\infty)}$$

A_s = Surface Area

T_s = Surface Temp

T_∞ = ambient Temp

where 'h' is a proportionality constant is called convective Heat transfer co-efficient.

$$\boxed{h = \frac{Q}{A_s \Delta T}} \Rightarrow \boxed{h = \frac{W}{m^2 K}}$$

→ 'h' is a not property of the Fluid, it is an experimentally determined parameter, whose value depends up on various Factors.

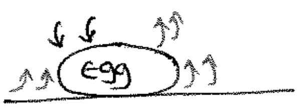
1) surface geometry
2) Nature of Fluid Motion
3) properties of Fluid
4) Bulk Fluid velocity.

→ convection Mechanism involving phase changes leads to the important fields of boiling [Evaporation] and condensation

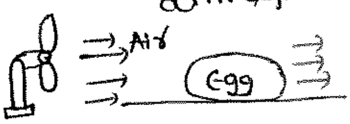
→ convection is of two Types

1) Free convection — caused by Buoyancy Effect, due to natural density difference., that's why it is also called as natural convection

2) Forced convection — Heat transfer by some external agency applied ~~like~~ for Fluid flow caused by pump, fan (or) by atmospheric winds.



Free convection



Forced convection

Conventional questions :-

1) A 2m-long, 0.3 cm diameter electrical wire extends across a room at 15°C, as shown in fig. Heat is generated in the wire as a result of resistance heating, and the surface temp of the wire is measured to be 152°C in steady operation. Also, the voltage drop and electric current through the wire ~~is~~ are measured to be 60V & 1.5 A, respectively. Disregarding any heat transfer by radiation, ~~at~~ determine the convection heat transfer co-efficient for heat transfer b/w the outer surface of the wire and the air in the room

Data :-

Length of wire $(L) = 2m$.

Dia of wire $(d) = 0.3cm = 0.003m$.

Ambient temp $(T_\infty) = 15°C = 288K$.

surface temp $(T_s) = 152°C = 425K$.

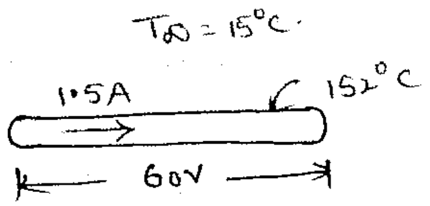
Voltage Drop $(V) = 60V$

current $(I) = 1.5A$.

Find :-

1) Find convective heat transfer co-efficient $(h) = ?$

schematic :-



Assumptions :-

1) steady state condition

2) one Dimensional convection ~~through out~~ the wire

3) Radiation heat transfer is negligible.

Sol:-

→ Here what ever the Electrical Energy is ~~too~~ supplied. that was converted into heat energy. & There no loss of energy while conversion means, total E → Q.

$$\dot{Q} = \dot{E} = VI$$

$$= 60 \times 1.5$$

$$= 90\ W$$

→ According to Newton's law of cooling.

$$\dot{Q}_{conv} = hA_s \Delta T$$

$$= hA_s (T_s - T_\infty)$$

$$\therefore h = \frac{\dot{Q}_{conv}}{A_s (T_s - T_\infty)} \qquad A_s = \pi D L$$

$$= \frac{90}{\pi \times 0.003 \times 2 (425 - 288)}$$

$$\therefore h = 34.9\ W/m^2 K.$$

2) A wire 10cm long & 1mm in diameter is held taud b/w two conducting supports in a water tank and is submerged. A controlled amount of current is made to pass through the wire until the temp of water becomes 100°C and its starts boiling. Make calculations for steady temp of wire if 23.5 watts of electric power is consumed. Take convective heat transfer co-efficient to be 5000 W/m² deg

Data

$L = 10cm = 0.1m$

$D = 1mm = 1\times10^{-3}$ m.

Here the water acts as a ambient & wire outside dia acts as a surface.

$\therefore$ ~~Ts =~~ $T_\infty = T_w = 100^\circ C = 373K$.

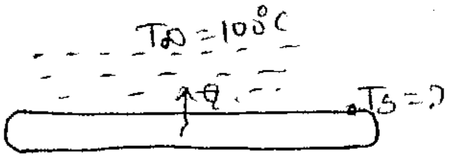
$\mathring{E} = \mathring{Q} = 23.5$ watt [Total Electrical Energy converted into Heat Energy]

$h = 500 W/m^2$-deg.

Find:-

1) surface temp of wire $(T_s) = ?$

Schematic:-



Assumptions:-

1) steady-state condition
2) one-Dimensional convection through the wire
3) Radiation heat transfer is negligible

Sol

$$\mathring{Q}_{conv} = hA\Delta t$$

$$= hA(T_s - T_\infty) \qquad A = \pi DL$$

$$23.5 = 500\times\pi\times \cancel{0.1}\ 1\times10^{-3}\times 0.1\times(\cancel{100}\ T_s - 100)$$

$$T_s = 115^\circ C$$

3) A 120W heater has been employed to maintain a plate of 0.25 m^2 area at a temp of 60°C, when the surroundings are at 20°C temp. What fraction of heat supplied is lost by natural convection? It may be preassumed that convection co-efficient conforms to the relation

$h = 2.5(\Delta T)^{0.25}$ W/m²K.

Data :-

Electrical Heater power (E) = 120W.

plate Area = 0.25 m^2

surface temp of plate (T_s) = 60°C. = 333 K

surrounding Temp (T_∞) = 20°C. = 293 K

$h = 2.5(\Delta T)^{0.25}$ W/m²K.

Find :-

1) How much heat is losted [supplied] by convection.

Assumptions :-

1) steady-state condition
2) one-Dimensional convection throughout the ~~heater~~ plate

Sol :-

$$\dot{Q} = hA\Delta T \qquad h = 2.5(\Delta T)^{0.25}$$

$$= 2.5(333-293)^{0.25}$$

$$Q = 6.287 \times 0.25 \times (333-293)$$

$$= 6.287 \text{ W/m}^2\text{K}$$

$$Q = 62.87 \text{ W}.$$

% of Heat Lost [supplied] by convection

$$= \frac{62.87}{120} = 52.39\%$$

[Remaining 47.61% is lost by Radiation]

3) Radiation :-

→ It is the Energy Emitted by Matter in the form of Electromagnetic waves as a Result of the changes in the Electronic configurations of atoms (or) Molecules.

→ Every substance above "0 K" Emitts Radiation.

→ Unlike conduction & convection, Radiation need not require a medium to heat transfer.

→ Heat transfer by Radiation is fastest [Equal to speed of Light ie 3,00,000 km/s) than both conduction & convection.

→ The Radiation can be affected through vaccume (or) a space devoid of any Matter.

→ Radiation exchange, in fact, occurs most Efficiently in vaccum.

→ In heat transfer we interested in Thermal Radiation not like x-rays, γ-rays, radio waves, Television waves.

→ The Thermal Radiation is limited to range of wave length b/w 0.1μ & 100μ of Electro magnatic spectrum

→ The Mechanism of the heat flow by Radiation consists of '3' distinct phases

1) conversion of Thermal Energy of the hot source into electromagnatic waves

2) passage of wave motion through Intervening space.

3) Transformation of waves in to heat.

→ It is Found from steafan - Boltzmann's law.

$$\boxed{E = \sigma A_s T_s^4} - W$$

σ = steafan boltzman const.

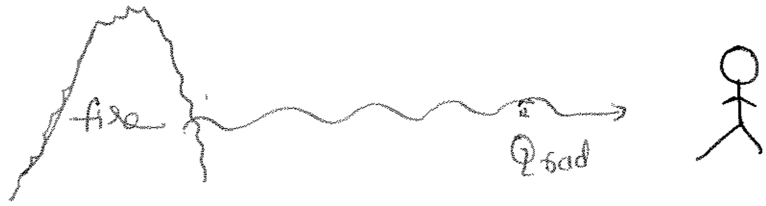
$= 5.67 \times 10^{-8}\ W/m^2K^4$

A = surface Area - m^2 ~~A~~ = such surface is called ideal Radiator (or) Black body.

T = Temparature.

Examples:-

1)



2)

→ The idealized surface that emitts radiation at this Max rate is called black body. & the radiation emitted by a block body is called black body radiation

→ The Radiation emitted by all real surfaces is lessthan the radiation emitted by a block body. at the same temparature, & it is expressed as

$$\boxed{\dot{Q}_{Emit} = \varepsilon \sigma A_s T_s^4} - W.$$

→ where ε is the emissivity of the surface.

→ The property emissivity, whose value ~~dep~~ in the range of $0 \le \varepsilon \le 1$. & it is depends upon surface characteristics and temparature.

→ It indicates how effectively the surface emits radiation compared to an ideal (or) black body radiator.

→ For a black body $\varepsilon = 1$.

→ Another important radiation property of a surface is its absorptivity (α)

→ It is nothing but the the body absorbed by the surface.

→ it value is in the range of $0 \leq \alpha \leq 1$.

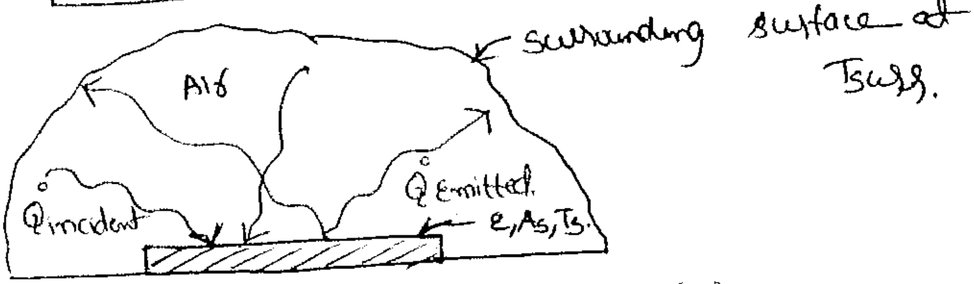
→ A black body absorbs more Radiation, & it is a perfect absorber & it value $\alpha = 1$

→ so black body acts as a perfect perfect absorber as well as perfect emitter and fastest.

→ Both ε & α of a surface depends upon the temp and the wave length of the Radiation

→ If Any body. & surroundings, The net rate of ~~Radiant~~ Radiation heat transfer b/w these two surfaces is given by.

$$\boxed{\dot{Q}_{rad} = \varepsilon\sigma A_s (T_s^4 - T_{surr}^4)} \leftarrow W$$



$$\dot{Q}_{rad} = \varepsilon\sigma A_s (T_s^4 - T_{surr}^4).$$

$$\boxed{Q_{incident} = Q_{absorbent}}$$

Conventional Questions :-

1) A Radiator in a domestic heating system operates at a surface temp of 60°C. calculate the heat flux at the surface of the radiator if it behaves as a black body.

Data :-

surface Temparature $(T_S) = 60^\circ C = 333\ K$.

Find :-

1) Heat Flux $(q) = ?$

Assumptions :-

1) steady ~~operating~~ state condition.
2) Heat transfer by convection is not considered
3) The surrounding surfaces are at a uniform temparature.

Sol :-

$$\dot{Q}_{rad} = \varepsilon \sigma A_S T_S^4.$$

$$\frac{\dot{Q}_{rad}}{A} = \varepsilon \sigma T_S^4$$

$$\dot{q} = 5.67 \times 10^{-8} \times 333^4$$

$$\dot{q} = 697.2\ W/m^2.$$

2) A cylindrical rod, 1.5m long 2cm in diameter, is heated Electrically and positioned in a vaccum furnace which has interior walls at 800K temp. A controlled amount of current is passed through the rod and its surface is maintained at 1000K. calculate the power supplied to the heating rod if its surface has an emissivity of 0.9

Data :-

$L = 1.5 m$

$D = 2 cm = 2 \times 10^{-2} m.$

$T_{surr} = 800K$

$T_s = 1000K.$

$\varepsilon = 0.9.$

Find :-

1) power supplied to the Heating. = ?

Assumptions :-

1) steady-state condition
2) Heat transfer by convection is not considered
3) The surrounding surfaces are at uniform temperature.

Sol :-

$$\dot{Q}_{rad} = \varepsilon \sigma A_s (T_s^4 - T_{surr}^4) \quad \Big| \quad A_s = \pi D L$$

$$= 0.9 \times 5.67 \times 10^{-8} \times \pi \times 2 \times 10^{-2} \times 1.5 (1000^4 - 800^4)$$

$$= 2838 \text{ W}.$$

∴ what ever the Heat radiated is by the power supplied,

∴ The power supplied must be 2838W.

3) considse a person standing in a room maintained at 22°c at all times. The inns suface of the walls, floors and ceiling of the house are observed to be at an avarage temp of 10°C in winter & 25°c in summer. Determine the Rate of radiation heat transfs b/w this person and the surrounding surfaces if the exposed suface area and the avarage suface temp of the person are $1.4 m^2$ and 30°c respectively.

Data :-

T_{surr} in winter $= 10°C = 283K.$

T_{surr} in summer $= 22°C = 295K$

$T_s = 30°C = 303K.$

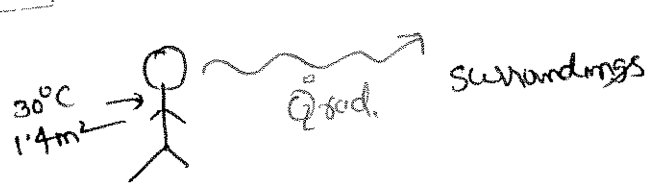
$A_s = 1.4 m^2.$

Find :-

1) $\dot{Q}_{rad}$ in winter $= ?$

2) $\dot{Q}_{rad}$ in summer $= ?$

schematic :-



Assumptions :-

1) steady state conditions
2) Heat transfs by convection is not considered
3) The surrounding sufaces are at a uniform temp.

Sol:-

1) $\dot{Q}_{rad}$ in winter $= \varepsilon \sigma A_s [T_s^4 - T_{surr, winter}^4]$

From the data, the emissivity of Human skin

$\varepsilon = 0.95$

$= 0.95 \times 5.67 \times 10^{-8} \times 1.4 [303^4 - 283^4]$

$= \underline{\underline{152 \ W}}$

2) $\dot{Q}_{rad}$ in summer $= \varepsilon \sigma A_s [T_s^4 - T_{surr, summer}^4]$

$= 0.95 \times 5.67 \times 10^{-8} \times 1.4 [303^4 - 295^4]$

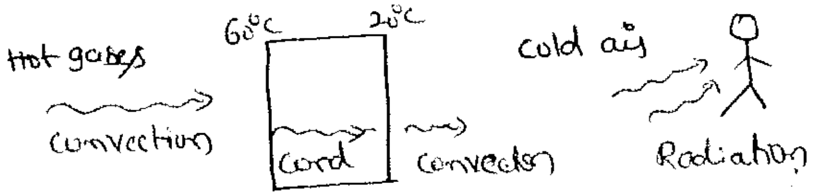
$= \underline{\underline{40.9 \ W}}.$

Combined modes of Heat transfer :-

Ex :-

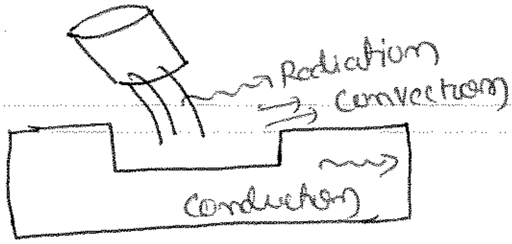
1) plan slab is exposed to Hot gases :- ~~the heat~~ ~~the~~ heat transfer will be Combined effect of

conduction + convection + Radiation



2) In Hot casting taken out from the Mold ~~and~~ kept In surrounding air will loose heat by convection & Radiation

3) When a molten metal is poured in a mould cavity heat transfer is occurred by conduction, convection and radiation.



conventional questions :-

1) consider a person standing in a breezy room at 20°C. Determine the total rate of heat transfer from this person if the exposed surface area and the average outer surface temp of the person are 1.6m² and 29°C, respectively and the convection heat transfer co-efficient is 6W/m²K.

Data :-

$T_{surr} = 20°C = 293K$

$A_s = 1.6m^2$

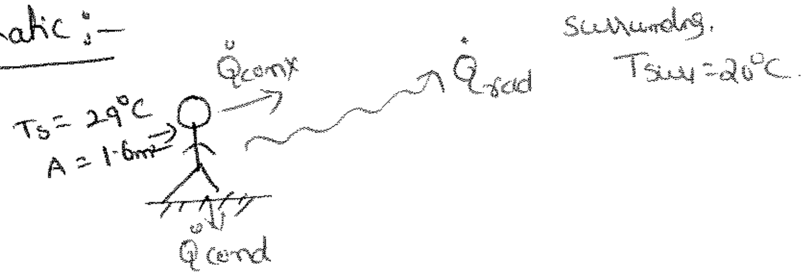
$T_s = 29°C = 302 K$

$h = 6W/m^2K$

Find :-

1) Total Heat rate =?

schematic :-



Assumptions :-

1) steady - state conditions
2) Heat conduction to the floor through the feet is negligible
3) The surround ~~temp~~ surfaces are at const temp.

Sol :-

$$\dot{Q}_{Total} = \dot{Q}_{conv} + Q_{rad}$$

$$\dot{Q}_{conv} = hA_s (T_s - T_\infty)$$

$$= 6.0 \times 1.6 (302 - 293)$$

$$= 86.4 \, W.$$

$$\dot{Q}_{rad} = \varepsilon \sigma A_s (T_s^4 - T_{surr}^4)$$

$$= 0.95 \times 5.67 \times 10^{-8} \times 1.6 \times (302^4 - 293^4)$$

$\varepsilon = 0.95$ for human body

$$= 81.7 W.$$

$$\therefore \dot{Q}_{total} = 86.4 + 81.7$$

$$= \underline{\underline{168.1 \, W}}$$

2) consider steady state heat transfer b/w two large ~~plates~~ parallel plates at constant temp of $T_1 = 300K$ & $T_2 = 200K$ that are $L = 1cm$ apart, as shown in fig. Assuming the surface to be black body (emissivity $\varepsilon = 1$), determine the rate of heat transfer b/w the plates per unit surface area assuming the gap b/w the plates is.

a) Filled with atmospheric air b) Evacuated
c) Filled with urethane insulation d) Filled with super-insulation that has an apparent thermal conductivity of 0.00002 W/mK

Data :-

$T_1 = 300K$, $T_2 = 200K$ $L = 1cm$
$= 0.01m$

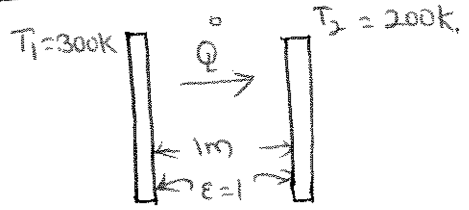
$\varepsilon = 1$ for black body.

$K_{super\ ins} = 0.00002\ W/mK$

Find :-

Find total Heat rate in All cases.

Schematic :-



Assumptions :-

1) steady-state condition
2) There are no natural convections current in ~~the~~ b/w plates
3) The surrounding Temp are at const.

Sol ~~1st case~~ 1st case :-

→ In this problem, There is a gap b/w the plates in first case of atmospheric air but there is no flowing of fluid so, what ever the heat transfer takes place that is pure conduction & Radiation [Because it is a black body with Emissivity is given]

∴

$$\dot{Q}_{tot} = \dot{Q}_{cond} + \dot{Q}_{rad}$$

$$\dot{Q}_{cond} = -KA\frac{dT}{dx}$$

$$= -\frac{0.0219 \times 1 \times (200-300)}{0.001}$$ Thermal conductivity of air (K_{air}) = 0.0219 W/m^2K

$$\dot{Q}_{cond} = \underline{219 W.}$$

$$\dot{Q}_{rad} = \varepsilon\sigma A_s (T_s^4 - T_{sur}^4)$$

Higher temp plate acts a surface & low temp plate acts a surroundings $\therefore T_s = 300K, T_{sur} = 200K$

$$\therefore \dot{Q}_{rad} = 1 \times 5.67 \times 10^{-8} \times 1 \times (300^4 - 200^4)$$
$$= 369 W.$$

$$\therefore \dot{Q}_{tot} = 219 + 369$$
$$= \underline{\underline{588W.}}$$

2nd case :-

→ In this case there is no Atmosphere air, means Evacuated. So, what ~~extra~~ ever Rate of Heat transfer takes place that is pure Radiation only.

$$\therefore \dot{Q}_{tot} = \dot{Q}_{rad}$$

$$\therefore \dot{Q}_{tot} = \dot{Q}_{rad} = \underline{369W.}$$

3rd case :-

→ In this cas the gap b/w plates are filled with urethan and it is acts a insulator, so no change of radiation & convection, what ever the heat

If the heat is conducted to the surface through a solid material of thermal conductivity 10W/mK., determine the temp gradient at the surface of the solid.

Data :-

$T_s = 200^\circ C = 473\,K.$

$T_{sur} = 50^\circ C = 323\,K.$

$h = 75\,W/m^2K.$

$\epsilon = 0.95$

$k = 10\,W/mK.$

Find :-

1) Temparature gradient. = ?

Assumptions :-

1) steady state condition

2) The surrounding temp are at constant

Sol :-

→ In this problem what eve the conduction happened in the solid plate is ~~convected~~ transfered to atmosphere by means of both convection and Radiation

∴ Heat conducted = Heat convected + Heat Radiated.

Ans
$\therefore \frac{dT}{dx} = -1336\,K/m$

$$-kA\frac{dT}{dx} = hA(T_s - T_\infty) + \epsilon\sigma A_s(T_s^4 - T_{sur}^4)$$

$$-10 \times 1 \times \frac{dT}{dx} = 75 \times 1 \times (473-323) + 0.95 \times 5.67 \times 10^{-8} \times 1 \times (473^4 - 323^4)$$

∴

transfer takes place that is pure conduction only.

$$\therefore \dot{Q}_{total} = \dot{Q}_{cond}$$

$$\dot{Q}_{cond} = -KA\frac{dT}{dx}$$

for urethane K = 0.026 W/mK.

$$= -0.026 \times 1 \times \frac{(200-300)}{0.01}$$

$$= \underline{\underline{260\ W}}$$

~~case~~ 4th case :-

→ In this case urethane ~~is~~ has replaced with super insulator of thermal conductivity K = 0.00002 W/mK

$$\therefore \dot{Q}_{total} = \dot{Q}_{cond}$$

$$\dot{Q}_{cond} = -KA\frac{dT}{dx}$$

$$= -0.00002 \times 1 \times \frac{(200-300)}{0.01}$$

$$= \underline{\underline{0.2\ W}}$$

Conclusion :-

→ with less thermal conductivity the heat transfer rate will decrease & it is an insulators.

3) A surface at 200°C losses heat both by convection and radiation to the surroundings at 50°C. The convection co-efficient is 75 W/m²K and the radiation factor due to geometric location and emissivity is 0.95

overal view of conduction :-

→ unlike temp, heat transfer has direction as well as Magnitude and it is a vector quantity

→ so must specify both direction and Magnitude inorder to describe heat transfer completly at a point.

50°C → only Magnitude

→ 80W/m²

↑ Magnitude & direction of heat flux at the same point.

Q = heat transfer

$\frac{Q}{A} = q =$ heat flux $= \frac{\text{heat transfer}}{\text{unit area}}$

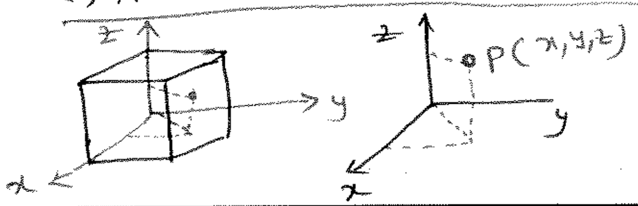
→ large the temp difference, large the heat transfer

→ The specification of the temp at a point in a Medium first requires the specification of the Location of that point

→ This can be done by choosing a suitable co-ordinate system such as rectangular, cylindrical (or) spherical coordinates depending up on geometry involved and convenient reference point (origin)

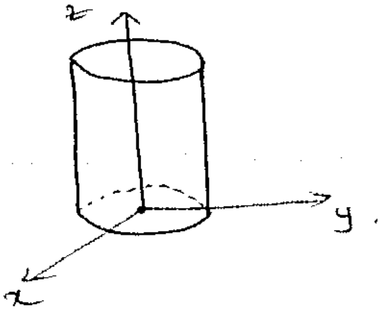
→ The Location of a point is specified as (x, y, z) in Rectangular coordinates, (r, ϕ, z) in spherical coordinates, (r, ϕ, θ) as in spherical coordinates.

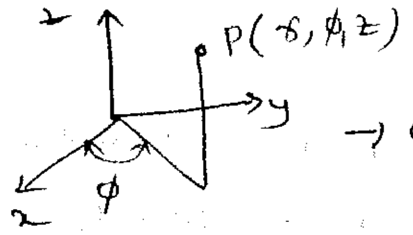
1)



→ Rectangular coordinates
(3P → PPP)

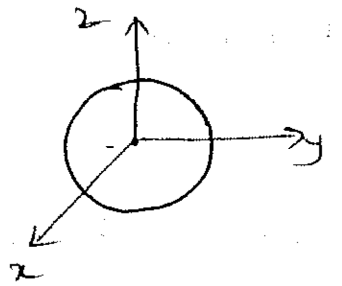
2)

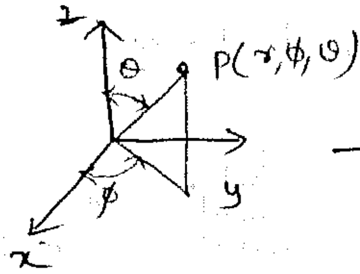




→ cylindrical coordinates
[2PR → PPR]

3)





→ spherical coordinates
[P2R → PRR]

→ Heat transfer is classified as also in conductions is
1) steady state ~~condution~~ conduction 2) unsteady state conduction

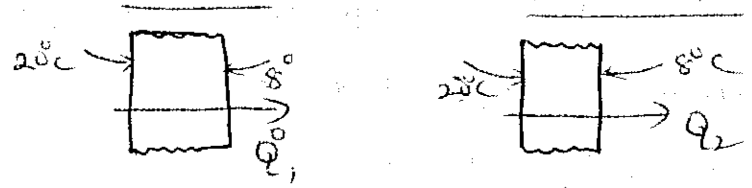
→ steady steady heat conduction → $\left(\frac{\partial T}{\partial r} = 0\right)$

→ unsteady steate (or) transient heat conduction $\left[\frac{\partial T}{\partial r} \neq 0\right]$

1) steady state conduction :-

→ The temp (or) heat transfer remains unchanged with time during steady heat transfer throughout a medium at any Location, although both quantities may varry from Location ~~Loc~~ to Location

Time = 12 PM Time = 2 pm

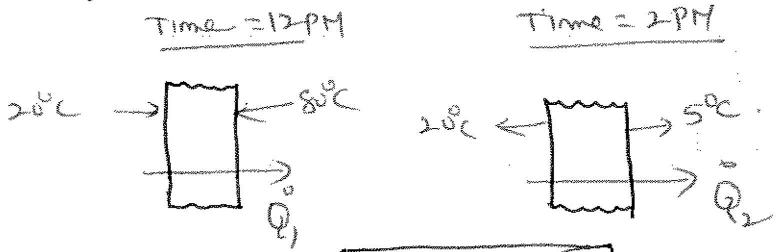


$$\boxed{\dot{Q}_1 = \dot{Q}_2}$$

→ Even the temp of Inner and outer ared different but at that time, ~~difference~~ Heat transfer both side are same.

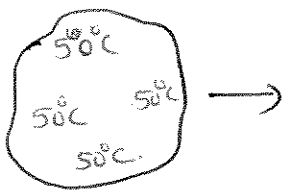
2) Transient heat conduction :-

→ During transient heat transfer, the temp normally varies with time as well as position



$$\boxed{\dot{Q}_1 \neq \dot{Q}_2}$$

→ In special case a variation with time, but not with position, the temp of Medium changes uniformly with time, such system are called Lumped system



→ Roast Meat.

A small Roast meat coming out of an oven, measurement indicates that the temp of the Roast changes with time but it does not changes much with position at any given time.

→ Most heat transfer proplems encounted in practice are Transient in nature but they are usually analyzed under some pres assumed steady conditions.

→ since steady processes are easy to analyze.

→ Heat transfer problems also classifies as Dimensionality ie; - -

Multi Dimensional Heat transfer :-

→ Heat transfer problems are also classified as being
1) one - Dimensional [1-D]
2) Two - Dimensional [2-D]
3) Three - Dimensional [3-D]

→ In most general cases, heat transfer through a medium is 3-Dimensional

→ The temp distribution through~~out~~ out the Medium at a specified time as well as the heat transfer rate at any location in this general case can be described by a set of 3-coordinates.
1) x, y, z in the rectangular (or) cartesian coordinate syst
2) r, ϕ, z → in the cylindrical coordinate system
3) r, ϕ, θ → in spherical (or) polar coordinate system.

→ The Temp distributions are in above cases are
1) $T(x, y, z, t)$ 2) $T(r, \phi, z, t)$ 3) $T(r, \phi, \theta, t)$

→ The Temp in a Medium, in some cases varies mainly in two primary directions, variation of temp in 3rd direction is negligible ie - - called Two-Dimensional heat transfer

Example :- steady state temp distribution in a long bar of rectangular cross section can be expressed as $T(x,y)$, temp variation in z - direction is negligible and there is no change with time.

→ Heat transfer problems is said to be one-Dimensional if temp in the Medium varries in one direction only. and other directions are negligible.

Examples:-

1) Heat transfer through a glass of window
↓
1^r direction only [normal direction]

2) Hot water passing through a pipe
↓
Radial direction only.

3) Egg Dropped in a boiling water
↓
Radial direction to Mid point of Egg.

→ Heat transfer problems also classified as - - - Heat Generation concept i.e.

Heat Generation :-

→ A Medium through out which heat is conducted may involve the conversion of Mechanical, Electrical, Nuclear (or) chemical energy in to heat Energy.

→ In Heat conduction analysis such conversion process are characterised as Heat [Thermal Energy] generation.

examples :-

1) When current passing in Resistance wire generates Heat $[I^2R]$.

2) Large amount of heat is generated in the fluid element of Nuclear Reactor as a result of nuclear fission that serves as the heat source for power plant

3) Exothermic & Endo Thermic reactions of chemical process

4) Absorbtion of solar radiation by water can treated as Heat generation

→ It is a volumetric phenomenon and it occurs through the Medium so rate of Heat generation in Medium is usually specified per unit volume & denoted by $\dot{e}_{gen}$ (or) q_{gen} (or) Q_{gen}

Note :-

→ Heat Generation is expressed per unit volume ~~w/m~~. $[W/m^3]$

→ Heat Flux is expressed per unit surface area $[W/m^2]$.

conventional Questions :-

1) The Resistance wire of 1200 W hair dryer is 80 cm long and has a diameter of D = 0.3 cm ; Determine the rate of heat generation in the wire per unit volume and heat flux on the outer surface of wire as a result of this heat generation

Data :-

$\overset{\circ}{E} = 1200W$.

$L = 80cm = 80\times10^{-2} m$

$D = 0.3cm = 0.3\times10^{-2} m$

Find :-

1) Heat generation ($\overset{\circ}{e}_{gen}$) = ?

2) Heat flux (q) = ?

Assumptions :-

1) steady state condition

2) 2D Heat flow [Because Heat generation is per unit volume means two perameters involved Length (L) & Diameter (D).

3) Heat generated uniformly through the wire.

sol :-

1) $$\overset{\circ}{e}_{gen} = \frac{\overset{\circ}{E}}{V_{wire}} = \frac{1200}{\frac{\pi}{4}\times D^2\times L} = \frac{1200}{\frac{\pi}{4}(0.3\times10^{-2})^2\times 80\times10^{-2}}$$

$$\therefore \overset{\circ}{e}_{gen} = 2.12\ w/m^3$$

2) $$q = \frac{\overset{\circ}{E}}{A_{wire}} = \frac{1200}{\pi DL} = \frac{1200}{\pi\times0.3\times10^{-2}\times80\times10^{-2}}$$

$$q = 15.9 w/m^2$$

Fourier's law of Conduction :-

Assumptions :-

1) Steady-State Heat transfer (properties donot change with time)
2) One-Dimensional Heat transfer (particular direction)
3) Material is Homogenious (constant density)
4) The bounding surfaces are Isothermal.

According Fouries law.

$$\dot{Q} \propto A \frac{dT}{dx}$$

$$\boxed{\dot{Q} = -kA \frac{dT}{dx}}$$

where

$\dot{Q}$ = Heat transfer Rate — W

A = surface area — m^2

$\frac{dT}{dx}$ = Temparature gradient [-ve in direction] — m/k

k = Thermal conductivity — W/mK.

→ The heat transfer rate is normal to a surface of constant temparature, called as isothermal thermal temparature.

→ For cartesian coordinates, the general expression for Heat transfer rate in 3-D

$$\boxed{\dot{Q} = -KA \frac{dT}{dx}, \quad \dot{Q} = -kA \frac{dT}{dy}, \quad \dot{Q} = -kA \frac{dT}{dz}}$$

Thermal conductivity (k) :-

→ It is a not constant & it is found from experiments

→ It is numerically equal to heat transfer through an area of $1 m^2$ of a slab of 1m thickness when two faces are maintained at a temp difference of $1°C$ (or) 1k. It's unit is $w/m\text{-}k$ (or) $w/m°c$.

→ 'k' actually Represents the ability of the Material to conduct heat ie how fast heat will flow in material. If the thermal conductivity will large the heat transfer rate will be large.

Difference b/w spl heat & Thermal conductivity :-

→ Thermal conductivity represents the ability of the medium to transfer heat, where as spl heat represents the ability of the medium to storage (or) absorb heat.

Iron	water
$k = 80.6 \frac{w}{mk}$	$k = 0.608\ w/mk$
$c = 0.45\ kJ/kgk$	$c = 4.18\ kJ/kg.k$

→ Though iron is very good conductor of heat, but it is bad storage of heat compare to water, where as water is good absorbs of heat but bad conductor of heat.

→ From experiments

$$\boxed{K_{solids} = K_0 (1 + \beta t)}$$

where

K_{solid} = Thermal conductivity at any temp 'T'

K_0 = Thermal conductivity at 0°C

β = co-efficient which depends on material

$$\boxed{K_{liquids} \propto \rho^{4/3}}$$

$$\boxed{K_{solids} > K_{liquids} > K_{gases}}$$

Note :- A good conductor of Electricity is also a good conductor of Heat, but a good conductor of heat need not to be a good conductor of electricity

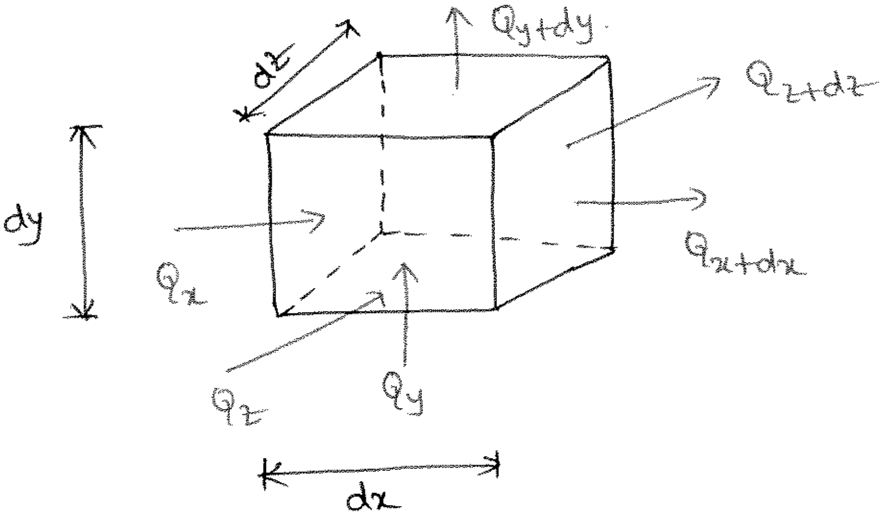
ex:- Diamond.

General Heat conduction Equation :-

→ Heat conduction is said to be Multidimensional & in this section we develop the governing differential equation in such system in Rectangular, cylindrical, and spherical coordinate system.

1) Rectangular coordinates :-

Let us consider an element as show in fig.



→ A Differential volume Element with thickness dx, dy, and dz in x, y & z-directions respectively.

→ The volume of the element (V) = dx.dy dz.

→ The Rate of incoming and outgoing energy by conduction in respective directions are as show in above;

∴ according to Fouriers heat conduction Equation

$\cancel{q_x = -K_x\, dy\, dz}$

$$Q = -KA\frac{dT}{dx}$$

$\therefore \quad Q_x = -K_x dy dz \cdot \frac{\partial T}{\partial x}$ $\quad$ $\cancel{Q_{x+dx} = Q_x}$

Incoming $\quad$ outgoing $\cancel{Q_x + dx}$

$Q_y = -K_y dx dz \frac{\partial T}{\partial y}$ $\quad$ Q_{x+dx}

$Q_z = -K_z dx dy \frac{\partial T}{\partial z}$ $\quad$ Q_{y+dy}

Q_{z+dz}

The outgoing equations as per Taylor's series approximation and rearranging we get

$$Q_{x+dx} = Q_x + \frac{\partial}{\partial x}(Q_x) \cdot dx$$

$$Q_{y+dy} = Q_y + \frac{\partial}{\partial y}(Q_y) \, dy.$$

$$Q_{z+dz} = Q_z + \frac{\partial}{\partial z}(Q_z) \cdot dz.$$

→ From energy balance equation we have

$$E_{in} - E_{out} + E_{gen} = E_{control\ volume}$$

$$\Rightarrow Q_x + Q_y + Q_z - Q_{x+dx} - Q_{y+dy} - Q_{z+dz} + E_{gen} = E_{control\ vol}$$

→ Heat generation (q_{gen}) is in W/m^3 & E_{gen} is the Energy & it is in watt (W).

$$E_{gen} = q_{gen} \times vol$$

$$= q_{gen} \times dx \cdot dy \, dz.$$

→ $E_{cont\,vol}$ = energy accumulated (or) stored in control volume i.e as Heat in watts (W)

$$\therefore Q = mC_p dT.$$

$\Rightarrow Q_x + Q_y + Q_z - Q_{x+dx} - Q_{y+dy} - Q_{z+dz} + q_{gen}\, dx\, dy\, dz = mc_p \frac{dT}{dt}$

$\Rightarrow \cancel{Q_x} + \cancel{Q_y} + \cancel{Q_z} - \cancel{Q_x} - \frac{\partial}{\partial x}(Q_x)\, dx - \cancel{Q_y} - \frac{\partial}{\partial y}(Q_y)\, dy - \cancel{Q_z} + \frac{\partial}{\partial z}(Q_z)\, dz$

$+ q_{gen} \cdot dx \cdot dy\, dz = mc_p \frac{dT}{dt}.$

$\rho = \frac{m}{V}$

$m = \rho V$

$\Rightarrow q_{gen}\, dx\, dy\, dz - \frac{\partial}{\partial x}(Q_x)\, dx - \frac{\partial}{\partial y}(Q_y)\, dy - \frac{\partial}{\partial z}(Q_z)\, dz$

$= \rho \cdot dx \cdot dy\, dz \cdot c_p \frac{dT}{dt}$

$\Rightarrow q_{gen} \cdot dx\, dy\, dz - \frac{\partial}{\partial x}\left(-k_x \cdot dy\, dz \cdot \frac{\partial T}{\partial x}\right) dx - \frac{\partial}{\partial y}\left(-k_y\, dx\, dz \cdot \frac{\partial T}{\partial x}\right) dy$

$- \frac{\partial}{\partial z}\left(-k_z\, dx \cdot dy \cdot \frac{\partial T}{\partial x}\right) dz = \rho c_p\, dx\, dy\, dz \frac{dT}{dt}$

$\Rightarrow \cancel{dx\, dy\, dz}\left(q_{gen} + \frac{\partial}{\partial x}\left(k_x \cdot \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \cdot \frac{dT}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{dT}{\partial z}\right)\right)$

$= \rho c_p\, \cancel{dx\, dy\, dz} \cdot \frac{dT}{dt}$

$$\Rightarrow \boxed{q_g + \frac{\partial}{\partial x}\left(k_x \cdot \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \cdot \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \cdot \frac{\partial T}{\partial z}\right) = \rho c_p \frac{dT}{dt}}$$

$\Downarrow$

This is the generalized differential Equation for conduction in cartesian coordinates for a Homogenious Material.

$\Rightarrow$ If the material is Isotropic ($k = \text{Const}$), then the properties are same in all directions

$\therefore k_x = k_y = k_z = k$

$$\Rightarrow \frac{\partial}{\partial x}\left(k\cdot\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\cdot\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\cdot\frac{\partial T}{\partial z}\right) + q_{gen} = \rho c_p \frac{dT}{dt}$$

$$\Rightarrow \boxed{k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + q_{gen} = \rho c_p \frac{dT}{dt}}$$

(or)

$$\Rightarrow \boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_{gen}}{k} = \frac{\rho c_p}{k}\cdot\frac{dT}{dt}}$$

(or)

$$\Rightarrow \boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha}\cdot\frac{dT}{dt}}$$

$\Downarrow$

Conditions for above equation:-

1) Material is Homogenious
2) Material is Isotropic
3) Thermal conductivity is Independent of temperature

Special cases :-

case 1) :- steady state :- (properties donot change with time) $dT/dt = 0$. ↳ Time '0'

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k} = 0.}$$

↳ poisson's Equation

case 2) :- steady state & No heat generation

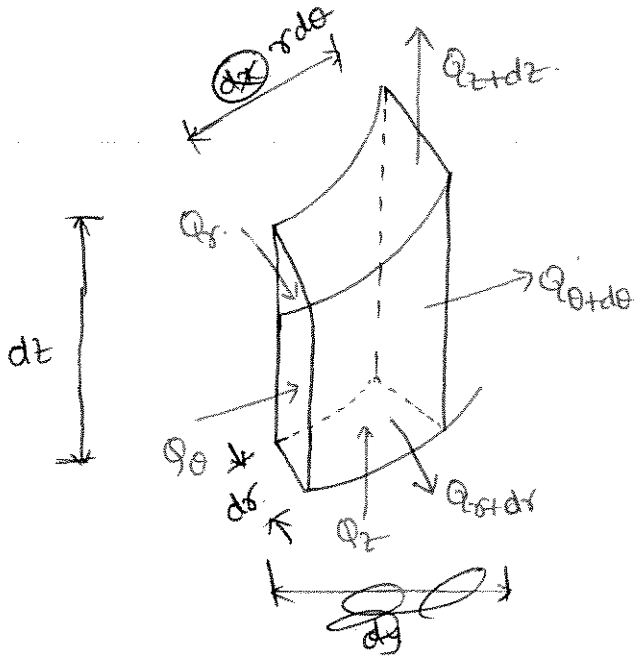
$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0} \rightarrow \text{Laplace equation}$$

Case 3) :- Transient (un steady) - No heat generation

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \cdot \frac{dT}{dt}}$$

2) cylindrical coordinates :-

Let us consider an Element as shown in fig.



In ~~spherical~~ cylindrical coordinates

~~x = rcosφ sinθ~~

The Equation can also obtained by transformation from rectangular coordinates using.

$x = r\cos\theta$
$y = r\sin\theta$.
$z = z$.

→ A Differential volume Element with thickness $dr, rd\theta$ & dz in r, θ, & z-directions

→ The volume of the Element $(V) = dr \cdot rd\theta \cdot dz$
(or)
$= r \cdot d\theta \cdot dr \cdot dz$.

→ The Rate of incomming and outgoing energy by conduction in respective directions are as shown in fig above

∴ According to Fourier's law of heat conduction

$$Q = -KA \frac{dT}{dx}$$

Incomming

$$Q_r = -K_r(r \cdot d\theta \cdot dz) \cdot \frac{\partial T}{\partial r}$$

$$Q_\theta = -K_\theta(dr \cdot dz) \frac{\partial T}{r \cdot \partial \theta}$$

$$Q_z = -K_z(r \cdot d\theta \cdot dr) \frac{\partial T}{\partial z}$$

out going

Q_{r+dr}

$Q_{\theta+d\theta}$

Q_{z+dz}.

$\downarrow$

According to Taylor's series approximation

$$Q_{r+dr} = Q_r + \frac{\partial}{\partial r}(Q_r)\, dr$$

$$Q_{r+d\theta} = Q_\theta + \frac{\partial}{r \cdot \partial \theta}(Q_\theta) \cdot r\, d\theta.$$

$$Q_{z+dz} = Q_z + \frac{\partial}{\partial z}(Q_z)\, dz.$$

→ From energy balance equation we have

$$E_{in} - E_{out} + E_{gen} = E_{control\ vol}.$$

$$\Rightarrow Q_r + Q_\theta + Q_z - Q_{r+dr} - Q_{\theta+d\theta} - Q_{z+dz} + E_{gen} = E_{C.V.}$$

→ Heat generation (q_{gen}) is in W/m^3 & ~~Energy~~ Egen is is in watts (W)

$$\therefore \quad E_{gen} = q_{gen} \times \text{Volume}$$

$$= q_{gen} = r \cdot d\theta \cdot dr \cdot dz.$$

$\rightarrow$ $E_{C.V}$ = Energy accumulated (or) stored in controlvolume

ie as Heat in watts (W)

$$\therefore Q = mc_p\, dT$$

$$\Rightarrow \cancel{Q_r} + \cancel{Q_\theta} + \cancel{Q_z} - \cancel{Q_r} - \frac{\partial}{\partial r}(Q_r)\, dr - \cancel{Q_\theta} - \frac{\partial}{r\cdot\partial\theta}(Q_\theta)\cdot r\,d\theta$$

$$- \cancel{Q_z} - \frac{\partial}{\partial z}(Q_z)\, dz + q_{gen}\cdot r\,d\theta\cdot dr\,dz = mc_p\frac{dT}{dt}$$

$$\Rightarrow q_{gen}\cdot r\,d\theta\cdot dr\,dz - \frac{\partial}{\partial r}(Q_r)\,dr - \frac{\partial}{r\cdot\partial\theta}(Q_\theta)\cdot r\cdot d\theta - \frac{\partial}{\partial z}(Q_z)\,dz$$

$$= mc_p\frac{dT}{dt}.$$

$$\Rightarrow q_{gen}\, r\,d\theta\cdot dr\cdot dz - \frac{\partial}{\partial r}\left(-k_r\cdot r\,d\theta\cdot dz\cdot\frac{\partial T}{\partial r}\right)dr - \frac{\partial}{r\cdot\partial\theta}\left(-k_\theta\cdot dr\,dz\,\frac{\partial T}{r\partial\theta}\right)r\,d\theta$$

$$-\frac{\partial}{\partial z}\left(-k_z\, r\cdot d\theta\cdot dr\cdot\frac{\partial T}{\partial z}\right)\cdot dz = \rho\times r\,d\theta\cdot dr\,dz\times c_p\frac{dT}{dt}$$

$$\Rightarrow \cancel{r\,d\theta\cdot dr\,dz}\left[q_{gen} + \frac{\partial}{\partial r}\left(k_r\frac{\partial T}{\partial r}\right) + \frac{\partial}{r\cdot\partial\theta}\left(k_\theta\cdot\frac{\partial T}{r\cdot\partial\theta}\right)\right)$$

$$+ \frac{\partial}{\partial z}\left(k_z\cdot\frac{\partial T}{\partial z}\right) = \rho\cdot\cancel{r\,d\theta\,dr\,dz}\times c_p\frac{dT}{dt}$$

$$\Rightarrow q_g + \frac{\partial}{\partial r}\left(k_r\cdot r\cdot\frac{1}{r}\cdot\frac{\partial T}{\partial r}\right) + \frac{\partial}{r\cdot\partial\theta}\left(k_\theta\cdot\frac{\partial T}{r\cdot\partial\theta}\right) + \frac{\partial}{\partial z}\left(k_z\cdot\frac{\partial T}{\partial z}\right)$$

$$= \rho c_p\frac{dT}{dt}$$

$$\Rightarrow \boxed{q_g + \frac{1}{r}\cdot\frac{\partial}{\partial r}\left(k_r\cdot r\cdot\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\cdot\frac{\partial}{\partial\theta}\left(k_\theta\frac{\partial T}{\partial\theta}\right) + \frac{\partial}{\partial z}\left(k_z\cdot\frac{\partial T}{\partial z}\right) = \rho c_p\frac{dT}{dt}}$$

That above equation is generalized differential equation for conduction in ~~cartesian~~ cylindrical coordinates for ~~Hogogenaly~~ Homogenious material.

=) If the material is Isotropic (K=const), then the properties are same in all directions

$$\therefore \quad K_r = K_\theta = K_z = K$$

$$\Rightarrow \boxed{K\left[\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\cdot\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\cdot\frac{\partial}{\partial \theta}\left(\frac{\partial T}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right)\right] + q_{gen} = \rho c_p \frac{dT}{dt}}$$

(d)

$$\Rightarrow \boxed{\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_{gen}}{K} = \frac{\rho c_p}{K}\frac{dT}{dt}}$$

(8)

$$\Rightarrow \boxed{\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{K} = \frac{1}{\alpha}\frac{\partial T}{dt}}$$

$\Downarrow$

Conditions for above equation:-

1) Material is Homogenious

2) Material is Isotropic

3) Thermal conductivity is indipendent of temparature.

Special cases :-

1) case 1) :- steady-state

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial T}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{K} = 0$$

2) case 2) :- steady-state No heat generation

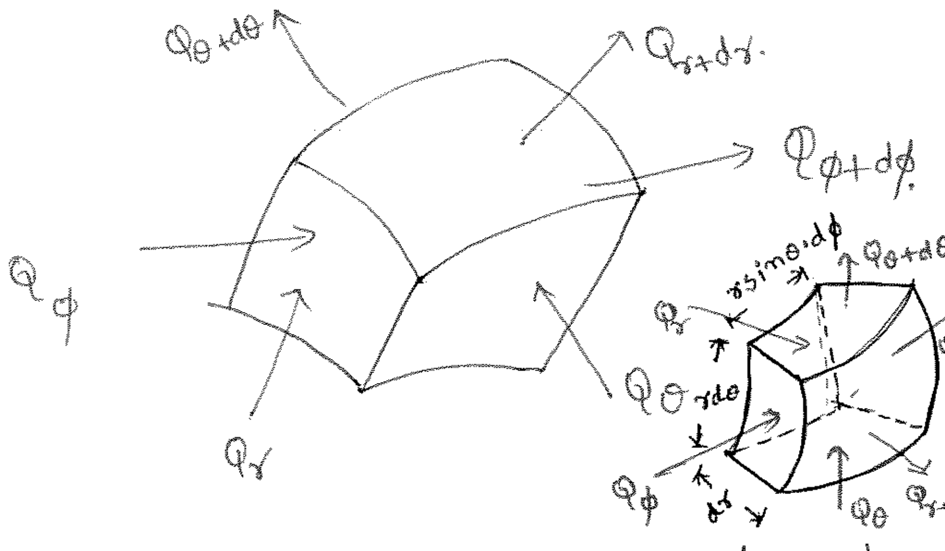
$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

case 3) :- Transient - no heat Generation

$$\frac{\partial^2 T}{\partial r^2} + \cancel{\partial^2 T} \; \frac{1}{r}\cdot\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{dT}{dt}.$$

3) Spherical coordinates :-

Let us consider an element as shown in fig.



In spherical coordinates, The Equation can also obtained by transformation from rectangular coordinates using

$x = r\sin\theta\cdot\sin\phi$

$y = r\sin\theta\cdot\cos\phi$

$z = r\cos\theta$.

→ A differential volume element with thickness dr, $r\,d\theta$ & $r\sin\theta\cdot d\phi$. ~~resp~~ in r, θ & ϕ directions respectively

→ The volume of the element $(V) = dr \times r d\theta \times r\sin\theta \, d\phi$

→ The Rate of incoming and outgoing energy by conduction in respective directions are as shown in fig above

∴ According to Fourier's law of heat conduction

$$\phi = -KA \cdot \frac{dT}{dx}$$

<u>Incoming</u> <u>outgoing</u>

$$Q_r = -K(r d\theta \times r\sin\theta \cdot d\phi)\frac{\partial T}{\partial r} \qquad Q_{r+dr}$$

$$Q_\theta = -K(dr \times r\sin\theta \cdot d\phi)\frac{\partial T}{r\partial\theta} \qquad Q_{\theta+d\theta}$$

$$Q_\phi = -K(dr \times r d\theta)\frac{\partial T}{r\sin\theta \cdot d\phi} \qquad Q_{\phi+d\phi}$$

According to Taylor's series approximation

$$Q_{r+dr} = Q_r + \frac{\partial}{\partial r}(Q_r) \cdot dr$$

$$Q_{\theta+d\theta} = Q_\theta + \frac{\partial}{r\partial\theta}(Q_\theta) \cdot r d\theta$$

$$Q_{\phi+d\phi} = Q_\phi + \frac{\partial}{r\sin\theta \, d\phi}(Q_\phi) \cdot r\sin\theta \cdot d\phi$$

→ From energy balance Equation we have

$$E_{in} - E_{out} + E_{gen} = E_{CV}$$

$$\Rightarrow Q_r + Q_\theta + Q_\phi - Q_{r+dr} - Q_{\theta+d\theta} - Q_{\phi+d\phi} + E_{gen} = E_{CV}$$

→ Heat generated (q_{gen}) is in W/m^3 & Energy is in watts (W)

$$\therefore E_{gen} = q_{gen} \times \text{volume}$$

$$= q_{gen} \times dr \times r d\theta \times r \sin\theta . d\phi .$$

→ $E_{c.v}$ = Energy accumulated (or) stored in control volume ie, as Heat in watts (W)

$$\therefore Q = m c_p \, dT$$

$$\Rightarrow \cancel{Q_r} + \cancel{Q_\theta} + \cancel{Q_\phi} - \cancel{Q_r} - \frac{\partial}{\partial r}(Q_r).dr - \cancel{Q_\theta} - \frac{\partial}{r\partial\theta}(Q_\theta) \cdot r d\theta$$

$$- \cancel{Q_\phi} - \frac{\partial}{r\sin\theta \, d\phi}(Q_\phi) . r\sin\theta . d\phi$$

$$+ q_{gen} \times dr \times r d\theta \times r\sin\theta . d\phi = m c_p \frac{dT}{dt}$$

$$\Rightarrow q_g \times dr \times r d\theta \times r\sin\theta . d\phi - \frac{\partial}{\partial r}(Q_r).dr - \frac{\partial}{r\partial\theta}(Q_\theta) . r d\theta$$

$$- \frac{\partial}{r\sin\theta \, d\phi}(Q_\phi) . r\sin\theta . d\phi = m c_p \frac{dT}{dt} .$$

$$\Rightarrow q_g \times dr \times r d\theta \times r\sin\theta \, d\phi - \frac{\partial}{\partial r}\left(-k . r d\theta \times r\sin\theta \, d\phi . \frac{\partial T}{\partial r}\right) \times dr .$$

$$- \frac{\partial}{r\partial\theta}\left(-k \times dr \times r\sin\theta . d\phi . \frac{\partial T}{r\partial\theta}\right) \times r . d\theta .$$

$$- \frac{\partial}{r\sin\theta \, d\phi}\left(-k . dr \times r d\theta . \frac{\partial T}{r\sin\theta . d\phi}\right) \times r\sin\theta . d\phi$$

$$= \rho \times dr \times r d\theta \times r\sin\theta . d\phi \times c_p \frac{dT}{dt}$$

$$\Rightarrow \cancel{dr} \times \cancel{r d\theta} \times \cancel{r}\sin\theta \cancel{d\phi} \left[q_g + \frac{\partial}{\partial r}\left(k \cdot \frac{\partial T}{\partial r}\right) + \frac{\partial}{r\partial\theta}\left(k \cdot \frac{\partial T}{r\partial\theta}\right) + \frac{\partial}{r\sin\theta \cdot d\phi}\left(k \cdot \frac{\partial T}{r\sin\theta \cdot d\phi}\right) \right] = \rho \times dr \times r d\theta \times r\sin\theta d\phi \times c_p \frac{dT}{dt}$$

$$\Rightarrow q_g + \frac{\partial}{\partial r}\left[k \cdot r^2 \times \frac{1}{r^2}\right] +$$

After lengthy manipulations finally we obtain

$$\boxed{q_g + \frac{1}{r^2}\frac{\partial}{\partial r}\left(k r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(k \cdot \sin\theta \cdot \frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}\left(k\frac{\partial T}{\partial\phi}\right) = \rho c_p \frac{dT}{dt}}$$

$\Downarrow$

This equation is ~~generally~~ generalized differential equation for conduction in spherical coordinates for homogeneous material

$\Rightarrow$ If the material is $\underset{(k = const)}{\underline{\text{Isotropic}}}$, then the properties are same in all directions

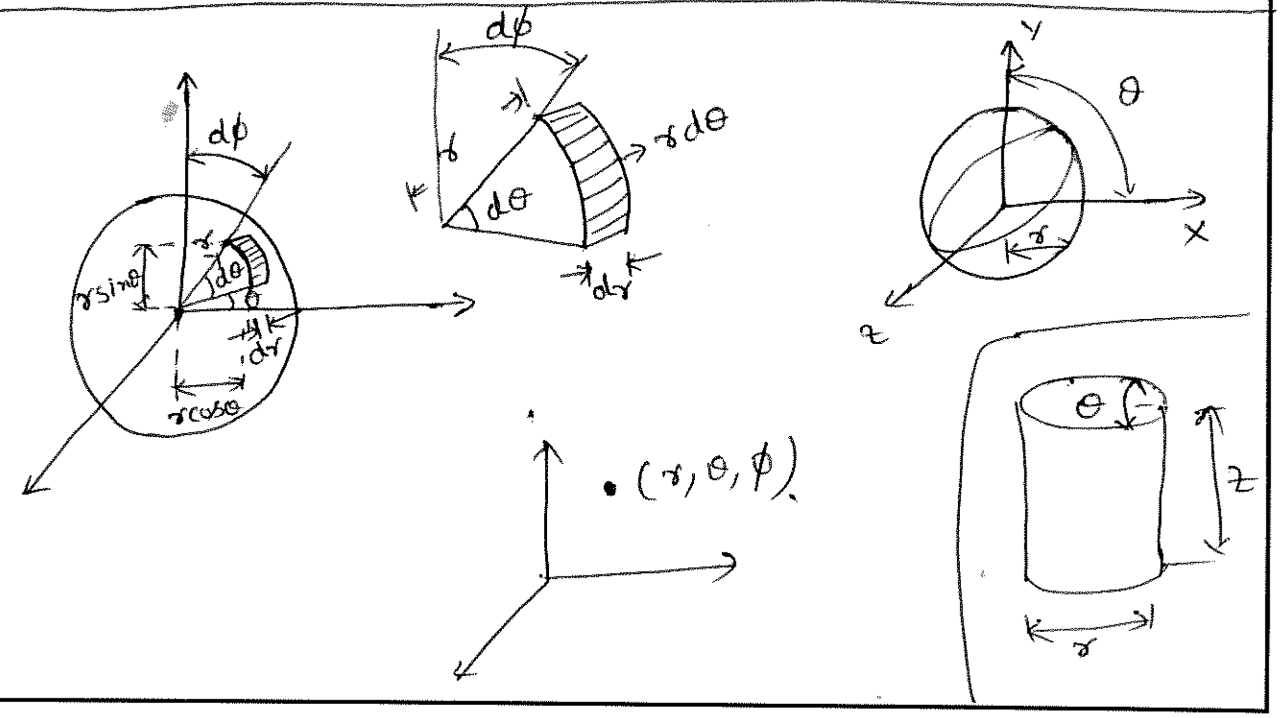
$$\therefore \quad K_r = K_\theta = K_\phi = K.$$

$$\Rightarrow \boxed{K\left[\frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta\cdot\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\cdot\frac{\partial}{\partial\phi}\left(\frac{\partial T}{\partial\phi}\right)\right]+q_g=\rho c_p\frac{dT}{dt}}$$

(8)

$$\Rightarrow \frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta\cdot\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}\left(\frac{\partial T}{\partial\phi}\right)+\frac{q_g}{K}=\rho c_p\frac{dT}{dt}.$$

$\Rightarrow$



One Dimensional Heat conduction Equation :-

→ Heat conduction in many geometries can be approximated as being one-Dimensional since heat conduction through these geometries is dominant in one direction and negligible in other directions.

→ If it is cartesian coordinates with one Dimensional ~~it is~~ developed a Large plane wall

→ If it is cylindrical coordinates with one Dimensional developed Long cylinder

→ If it is spherical coordinates with one Dimensional developed sphere

→ The combined 1-D Heat conduction equation is

**
***

$$\boxed{\frac{1}{r^n}\frac{\partial}{\partial r}\left(r^n \cdot k \cdot \frac{\partial T}{\partial r}\right) + q_g = \rho c_p \frac{dT}{dt}}$$

→ variable 'k'

**
***

$$\boxed{\frac{1}{r^n}\frac{\partial}{\partial r}\left(r^n \cdot k \cdot \frac{\partial T}{\partial r}\right) + \frac{q_g}{k} = \frac{1}{\alpha}\frac{dT}{dt}}$$

↳ constant 'k'

→ If $n = 0$ → It is a plane wall

$n = 1$ → It is a cylinder

$n = 2$ → It is a sphere

→ In plane wall 'r' is replaced with 'x'

Initial and Boundary conditions :-

→ The heat conduction equations ~~have~~ above were developed using an energy balance on a differential element inside the Medium, and they remain the same regardless of thermal conditions on the surfaces of the Medium

→ That is, the differential equations do not incorporate any information related to the conditions on the surface such as the surface temperature or a specified heat flux.

→ we know that now, that the heat flux and the temp distribution in a medium depends on the conditions at the surfaces, and the description of heat problems in a medium is not completly with out a full description of the thermal conditions at the bounding surface of the Medium.

→ The Mathematical expressions of the thermal conditions at the boundaries are called the "Boundary conditions"

→ For Mathematical point of view, solving a differential equation is essentially a process of removing derivatives, or an integration process, and thus the sollution of a differential equation typically involves arbitary constants.

→ But, since the differential equation has no place for the additional information of conditions, we need to supply them separately in the form of boundary (or) initial conditions.

→ To describe a heat transfer problem completly, two boundary conditions must be given for each direction of the coordinate system along which heat transfer significant.

→ For 1-D problems we need ~~two~~ two boundary conditions

→ For 2-D problems – we need Four boundary conditions

→ For 3-D problems – we need six boundary conditions

Example :-

→ In the case of wall of house, we need to specify the conditions at two Locations Inner and outer surfaces of the wall in 1-D heat transfer problems

→ In the case of ~~parallelpi~~ a parallelepiped, we need to specify six boundary condition. one at each face when 3-D heat transfer problem

→ The temperature at any point on the wall at a specified time also depends on the conditions of the wall begining of heat conduction process. such conditions, ~~of the wall~~ which is usually at time t = 0, is called the "Initial conditions", which is a Mathematical Expression for temp distribution of the medium initially.

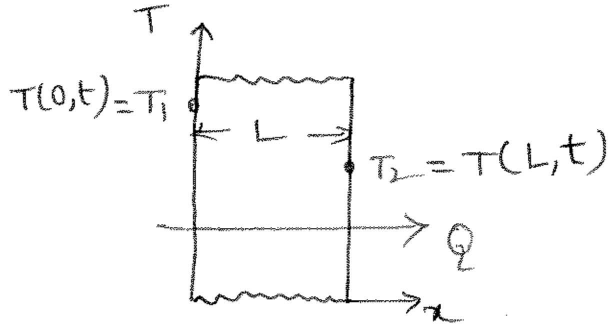
→ For steady state only we specify Initial conditions because for steady state time is not a consideration, but in unsteady state condition initial conditions need not to specify

→ There are Three kinds of Boundary conditions commonly appeared in Heat transfer are discussed below

1) prescribed temparature Boundary conditions
2) prescribed Heat flux Boundary conditions
3) convection Boundary conditions :- surface Energy Balance

1) prescribed Temparature Boundary conditions :-

→ A plane wall of thickness L, whose Left face ($x=0$) is maintained at uniform temp of T_1 and Right face at $x=L$ at uniform temp of T_2 as shown in fig. below



→ The Boundary conditions at two faces are

$$\text{At} \quad x=0,\ t=0 \rightarrow T(x,t) = T_1$$

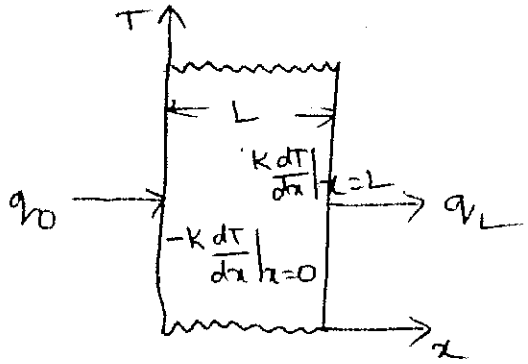
$$x=L,\ t=0 \rightarrow T(x,t) = T_2.$$

2) prescribed Heat Flux Boundary conditions :-

→ sometimes, the rate of heat transfer to a boundary is constent

ex:- an electrically heated surface, the rate of heat supply (capcity of heater) is const.

→ such conditions are called prescribed Heat flux Boundary condition. as show fig next



Heat Flux $q_x = -K \frac{dT}{dx}$.

at $x=0$, $q_x = q_0 =$ left face

$x=L$, $q_x = q_L =$ Right face

where

$$q_0 = -k\left\{\frac{dT}{dx}\right\}_{x=0}$$

and

$$q_L = -k\left\{\frac{dT}{dx}\right\}_{x=L}.$$

Special case :-

→ For Insulated boundary

$$q_x = 0 = -k\left\{\frac{dT}{dx}\right\}_x$$

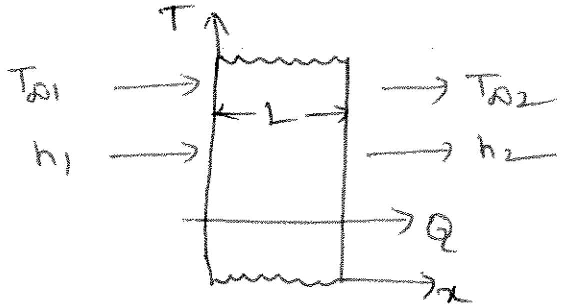
(or)

for Insulated surface $\left(\frac{dT}{dx}\right)_x = 0$.

where 'x' may be '0' for Left face (or) L for the right face depends upon boundary specified

3) Convection Boundary Conditions :- Surface Energy Balance

→ In most practical applications, the heat transfer dissipates by convection with a known value of heat transfer co-efficient 'h' at one (or) both boundary ~~fo~~ surface as shown in fig. below.



→ The Energy balance at any boundary surface i.e.

convection flux from the fluid surface = Heat flux conducted in to the boundary from the surface

At Left face i.e. $x=0$

$$h_1(T_{\infty 1} - T_{x=0}) = -k\left\{\frac{dT}{dx}\right\}_{x=0}$$

At Right face i.e, $x=L$

$$-k\left\{\frac{dT}{dx}\right\}_{x=L} = h_2(T_{x=L} - T_{\infty 2})$$

∴ All '3' above Boundary condition are for plane wall surface and we can write the boundary conditions similarly for cylinders and spheres.

conventional questions :-

1) The temparature distribution accross a wall, 1m thick at a certain instant of time is given as

$$T(x) = 900 - 300x - 50x^2$$

where 'T' is in degree celsius and x is in meters

The uniform heat generation of $1000\,W/m^3$ is present in wall area of $10m^2$ having the properties $\rho = 1600\,kg/m^3$, $k = 40\,W/mK$ and $c = 4\,kJ/kgK$.

1) Determine the rate of heat transfer entering the wall ($x=0$) and leaving the wall ($x=1m$)

2) Determine the rate of change of internal energy of the wall

3) Determine the time rate of temparature change at $x = 0, 0.5m$.

Data

$L = 1m$

$$T(x) = 900 - 300x - 50x^2$$

$q_g = 1000\,W/m^3$, $A = 10m^2$, $\rho = 1600\,kg/m^3$

$k = 40\,W/mK$, $C = 4\,kJ/kgK = 4000\,J/kgK$

Find :-

1) a) The Rate of heat transfer at $x = 0$. (Left face)
b) " " " " at $x = 1$ (Right face)

2) The Rate of change of Internal Energy

3) The Time rate of temp change at $x = 0$ and $x = 0.5m$ $\left(\frac{dT}{dt}\right)$

Assumptions :-

1) One Dimensional conduction in x-direction
2) Medium with constant properties [Homogenious Material throught]
3) Uniform internal heat generation

Sol :-

1).

$$T(x) = 900 - 300x - 50x^2$$

The temp gradient

$$\frac{dT}{dx} = -300 - 100x.$$

using boundary condition. the heat flux can find out

→ at Left face of the wall @ $x = 0$

$$q_{x=0} \quad @ \quad q = -k \cdot \frac{dT}{dx}$$

$$q_{x=0} = -k \left\{ \frac{dT}{dx} \right\}_{x=0}$$

$$= -k \left(-300 - 100x\right)_{x=0}$$

$$= -400 \left(-300 - 100(x)\right)$$

$$= 12,000 \text{ W/m}^2$$

→ ~~at Right of the wall at $x=1$~~

∴ Heat Leaving the Left face $= A \times q_{x=0}$

$$= 10 \times 12,000$$

$$= 1,20,000 \text{ W}$$

$$= 120 \text{ kW}$$

→ at Right of the face at $x = 1m$.

$$Q_{x=L} = -KA \cdot \left\{\frac{dT}{dx}\right\}_{x=L}$$

$$= -K \times A \left(-300 - 100x\right)_{x=L} \quad \Big| \, L = 1m$$

$$= -40 \times 10 \left(-300 - 100 \times 1\right)$$

$$= 1,60,000 \, W$$

$$= \underline{\underline{160 \, kW}}.$$

2) The Rate of change of internal Energy

= Rate of heat entering at the left face + Rate of heat generation − Rate of heat leaving right face

$$= Q_{x=0} + q \times Vol - Q_{x=L}$$

$$= 120 + q \times A \times L - 160$$

$$= 120 + 1 \times 10 \times 1 - 160$$

$$= \underline{\underline{-30 \, kW}}.$$

3) The Rate of change of temp in the wall can ~~be calculated~~
→ first generalized heat conduction equation complete

$$\frac{\partial}{\partial x}\left(K_x \cdot \frac{\partial T}{\partial x}\right) + q_g = \rho c_p \frac{dT}{dx} \rightarrow \text{for } \underline{\underline{1-D}}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) + \frac{q_g}{K} = \frac{\rho c_p}{K} \frac{dt}{dx} \quad \Big| \, \frac{\rho c_p}{K} = \frac{1}{\alpha}$$

$$\frac{dT}{dt} = \frac{K}{\rho c_p}\left[\frac{d}{dx}\left(\frac{dT}{dx}\right) + \frac{q_g}{K}\right]$$

$$= \frac{d}{dx}\left[\frac{dT}{dx}\right] = \cancel{\phi} \cdot \phi$$

$$= \frac{d}{dx}\left[-300 - 100x\right]$$

$$= -100.$$

$$\therefore \frac{dT}{dt} = \frac{40}{1600 \times 4000} \times \left[-100 + \frac{1000}{40}\right]$$

$$\frac{dT}{dt} = -4.6875 \times 10^{-4}\ {}^{\circ}C/s.$$

2) The temp variation in a slab is given by $T = 100 + 200x - 500x^2$ where x is in meters and T is $^{\circ}C$. $x = 0$ at the left face and $x = 0.3$ at the right face, the thermal conductivity of the material is $45 W/mK$, $C_p = 4 kJ/kgK$, $\rho = 1600 kg/m^3$

Determine

1) Temp at both surfaces
2) Heat transfer at the left face and it's direction
3) " " " Right " " "
4) Is there any heat generation in the slab, if so, how much
5) Max temp in the slab and it's location
6) Draw the temp profile
7) The rate of change of temp at $x = 0.1m$ if the heat generation is suddenly doubled

Data

$T = 100 + 200x - 500x^2$

$L = 0.3m$, $K = 45 W/mK$, $C_p = 4 kJ/kg K$
$= 4000 J/kgK$

$\rho = 1600 kg/m^3$

Find :-

1) $T_1 = ?$ $T_2 = ?$

2) $Q_{x=0} = ?$

3) $Q_{x=0.3} = ?$

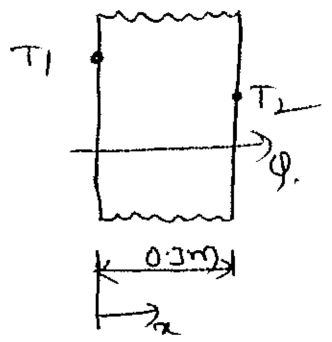
4) $q_g = ?$

5) $T_{max} = ?$ and ~~distance~~ $x = ?$ at T_{max}

6) Temp profile

7) $\frac{dT}{dt} = ?$ at $x = 0.1m$

schematic :-



Assumptions :-

1) one Dimensional Heat conduction in x-direction

2) Medium with const properties [Homogeneous Material through out]

3) ~~uniform~~ heat generation is different

1)
$$T = 100 + 200x - 500x^2$$

at $x = 0$.

$$T_1 = 100 + 200(0) - 500(0)$$

$$\therefore T_1 = 100^\circ C.$$

at $x = 0.3$

$$T_2 = 100 + 200(0.3) + 500(0.3)^2$$

$$T_2 = 115^\circ C.$$

2)
$$Q_{Left, x=0} = -KA \left.\frac{dT}{dx}\right|_{x=0}$$

$$T = 100 + 200x - 500x^2$$

$$\frac{dT}{dx} = 0 + 200 - 1000x.$$

$$\left.\frac{dT}{dx}\right|_{x=0} = 200.$$

$$\therefore q_{x=0} = -45 \times 1 \times 200$$

$$= -9 \times 10^3 \ w/m^2$$

$$q_{Left} = 9 \times 10^3 \ w/m^2 \ (\leftarrow \text{towards left})$$

3)
$$Q_{Right, x=0.3} = -KA \left.\frac{dT}{dx}\right|_{x=0.3}$$

~~$q_{x=0.3} = -45 \times 1 \times$~~

$$\left.\frac{dT}{dx}\right|_{x=0.3} = 200 - 1000(0.3)$$

$$= -100$$

$\therefore Q_{x=0.3} = -45 \times 1 \times -100$

$= 4.5 \times 10^3 \ \omega/m^2$ (→ Towards Right)

4) The generalyzed Heat conduction Equation for cartesian coordinates

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha} \cdot \frac{dT}{dt}.$$

Here the problems ~~is~~ in 1-D ie x-direction

$$\therefore \frac{\partial^2 T}{\partial x^2} + \frac{q_g}{k} = \frac{1}{\alpha} \frac{dT}{dt}$$

$$T = 100 + 200 - 500x^2$$

$$\frac{dT}{dx} = 0 + 200 - 1000x$$

$$\frac{d^2 T}{dx^2} = -1000.$$

Here we assume steady state $\therefore \frac{dT}{dt} = 0$

$$\therefore -1000 + \frac{q_g}{45} = \frac{1}{\alpha}(0)$$

$$q_g = 45 \times 10^3 \ \omega/m^3$$

5) $T = 100 + 200x - 500x^2$

for getting Max (or) Min value differentiate the given equation and equating with zero (0)

$$\therefore \frac{dT}{dx} = 200 - 1000x = 0$$

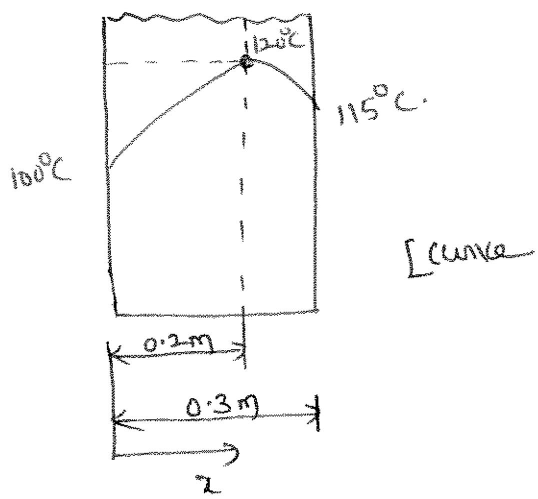
$$200 - 1000x = 0$$

$$1000x = 200 \Rightarrow \underline{\underline{x = 0.2\ m}}$$

$$\therefore T_{max} = 100 + 200(0.2) - 500(0.2)^2$$

$$T_{max} = 120^\circ C.$$

6) Temp profile



$T = 100 + 200x - 500x^2$ = curve

7) $\frac{\partial^2 T}{\partial x^2} + \frac{q_g}{k} = \frac{1}{\alpha} \cdot \frac{dT}{dt}$ [q_g = double

$\frac{dT}{dt}$ = ? at $x = 0.1m$

$$\cancel{\frac{d^2T}{dx^2}} - 1000 + \frac{2 \times 4.5 \times 10^3}{45} = \frac{45}{1600 \times 4000} \times \frac{dT}{dt}$$

$$\therefore \frac{d^2T}{dx^2} = -1000.$$

$$\therefore -1000 + \frac{2 \times 4.5 \times 10^3}{45} = \frac{45}{1600 \times 4000} \times \frac{dT}{dt}$$

$$\therefore \frac{dT}{dt} = 7.031 \cdot 10^{-3}\ °C/sec.$$

3) At a certain time, the temp distribution in a long cylindrical tube which a inner radius of 250mm and outside radius of 400mm is given by.

$$T(r) = 750 + 1000r - 5000r^2 \ ^\circ C.$$

where 'r' in meters. The Thermal conductivity and Thermal Diffusivity of the tube Material are 58W/mK and 0.004m²/hr respectively. calculate.

i) Rate of heat flow at inside and outside surface per unit length

ii) Rate of heat storage per unit length and

iii) Rate of change of temp at inner and outer surface

Data :-

· Hollow cylinder

$$T(r) = 750 + 1000r - 5000r^2 \ ^\circ c.$$

$$k = 58 W/mK, \quad \alpha = 0.004 m^2/hr.$$

$$r_1 = 250mm = 0.25m, \quad r_2 = 400mm = 0.4m$$

Find :-

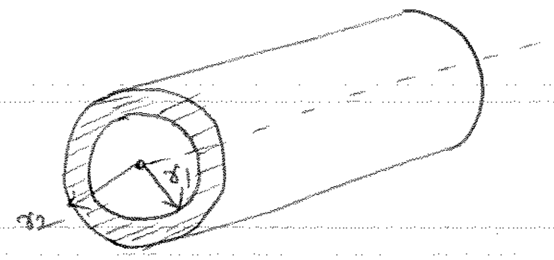
1) $\left[\frac{Q}{L}\right]_{r=r_1}$, $\left(\frac{Q}{L}\right)_{r=r_2}$

2) $\frac{Q}{L}$ Rate of heat store /unit length

3) Rate of change of temp at inner & outer surface

$$\left(\frac{dT}{dt}\right)_{r=r_1} \ \& \ \left(\frac{dT}{dt}\right)_{r=r_2}$$

Schematic :-



Assumptions :-

1) Heat flow is in Radial direction only. [1-D]
2) constant properties [Homogenious Material]
3) no heat generation in the element.

Sol

1) $T(r) = 750 + 1000r - 5000r^2$ °C.

$$\frac{dT}{dr} = 1000 - 10,000\, r.$$

i) Rate of Heat transfer in inside surface ($r = r_1$)

$$\left[Q\right]_{r=r_1} = -kA\left[\frac{dT}{dr}\right]_{r=r_1}$$

$A = \pi D L$
$= 2\pi r L$

$$Q = -k \cdot 2\pi r_1 L \left|\frac{dT}{dr}\right|_{r=r_1}$$

$$\left|\frac{Q}{L}\right|_{r=r_1} = -k \cdot 2\pi r_1 \left|\frac{dT}{dr}\right|_{r=r_1}$$

$$\left|\frac{Q}{L}\right|_{r=r_1} = -58 \times 2 \times \pi \times 0.25\,(1000 - 10,000 \times 0.25)$$

$$\left|\frac{Q}{L}\right|_{r=r_1} = 13.66 \times 10^4 \text{ W/m.}$$

ii) Rate of Heat transfer in outside surface ($r = r_2$)

$$\left|\frac{Q}{L}\right|_{r=r_2} = -K \times 2 \times \pi \times r_2 \left|\frac{dT}{dr}\right|_{r=r_2}$$

$$= -58 \times 2 \times \pi \times 0.4 \,(1000 - 10{,}000 \times 0.4)$$

$$\left|\frac{Q}{L}\right|_{r=r_2} = 4.35 \times 10^5 \ \omega/m$$

2) Rate of heat storage per unit length

$$= \left|\frac{Q}{L}\right|_{r=r_1} - \left|\frac{Q}{L}\right|_{r=r_2}$$

$$= 13.66 \times 10^4 - 4.37 \times 10^5$$

$$= -3 \times 10^5 \ \omega/m.$$

3) Rate of change of temp at inner & outer surfaces

Rate of change of temp at inner & outer surfaces generalized Heat conduction equation for cylindrical coordinates

$$\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\cdot\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial T}{\partial \theta}\left(\frac{\partial T}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) + \frac{q_g}{K}$$

$$= \frac{1}{\alpha}\cdot\frac{dT}{dt}$$

Here Heat is Radial direction (1-D) and no heat generation

$$\therefore \frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\cdot\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\cdot\frac{dT}{dt}.$$

$$\frac{dT}{dt} = \frac{\alpha}{r}\cdot\frac{\partial}{\partial r}\left(r\cdot\frac{dT}{dr}\right)$$

$$T = 750 + 1000r - 5000r^2$$

$$\frac{dT}{dr} = 1000 - 10{,}000\,r.$$

$$\frac{d}{dr}\left(r \cdot \frac{dT}{dr}\right) = r\left(1000 - 10{,}000r\right)$$

$$= 1000r - 10{,}000r^2$$

$$\frac{d}{dr}\left(r \cdot \frac{dT}{dr}\right) = \frac{d}{dr}\left(1000 \cdot r - 10{,}000r^2\right)$$

$$= 1000 - 20{,}000r.$$

i) $\therefore$ $$\left|\frac{dT}{dt}\right|_{r=r_1} = \frac{\alpha}{r_1}\left(1000 - 20{,}000\, r_1\right)$$

$$= \frac{0.004}{0.25}\left(1000 - 20{,}000 \times 0.25\right)$$

$$= -64°c/hr.$$

ii) $$\left|\frac{dT}{dt}\right|_{r=r_2} = \frac{\alpha}{r_2}\left(1000 - 20{,}000 r_2\right)$$

$$= \frac{0.004}{0.4}\left(1000 - 20{,}000 \times 0.4\right)$$

$$= -70°c/hr$$